Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WMA12/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

Pearson

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1. Find the first four terms, in ascending powers of x, of the binomial expansion of

$$\left(2+\frac{3}{8}x\right)^{10}$$

Give each coefficient as an integer.

(4)

Binomial expansion formula :

$$(a+bx)^n = {}^nC_0\, a^n (bx)^0 + {}^nC_1\, a^{n-1}(bx)^1 + {}^nC_2\, a^{n-2}(bx)^2 + \dots$$

$${}^nC_r = \frac{n!}{(n-r)!\,r!}$$

n : power of binomial

r : (position of term) − 1

NOTE : • FIRST AND LAST ${}^nC_r$ ARE ALWAYS $= 1$

• WHEN $r = n$, then ${}^nC_r = n$

$\left(2+\frac{3}{8}x\right)^{10}$ becomes :

$${}^{10}C_0\, 2^{10}\left(\frac{3}{8}x\right)^0 + {}^{10}C_1\, 2^{10-1}\left(\frac{3}{8}x\right)^1 + {}^{10}C_2\, 2^{10-2}\left(\frac{3}{8}x\right)^2 + {}^{10}C_3\, 2^{10-3}\left(\frac{3}{8}x\right)^3 + \dots$$

${}^{10}C_0 = 1$

${}^{10}C_1 = 10$

${}^{10}C_2 = \frac{10!}{(10-2)!\,2!} = 45$

${}^{10}C_3 = \frac{10!}{(10-3)!\,3!} = 120$

NOTE : • FIRST AND LAST ${}^nC_r$ ARE ALWAYS $= 1$

• WHEN $r = n$, then ${}^nC_r = n$

$$= (1)(1024) + 10(512)\left(\frac{3}{8}\right)x + 45(256)\left(\frac{9}{64}\right)x^2 + 120(128)\left(\frac{27}{128}\right)x^3$$

$$= 1024 + 1920x + 1620x^2 + 810x^3$$

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Question 1 continued

(Total 4 marks)

Q1

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2.

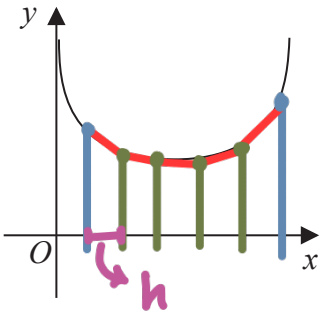

Figure 1

Figure 1 shows the graph of

$$y = 1 - \log_{10}(\sin x) \qquad 0 < x < \pi$$

where x is in radians. DON'T FORGET TO MAKE SURE CALCULATOR IS IN RADIANS!

The table below shows some values of x and y for this graph, with values of y given to 3 decimal places.

Interval btwn two x-coords = h of trapezium

x	0.5	1	1.5	2	2.5	3
y	1.319	1.075	1.001	1.041	1.223	1.850

y_1 y_6

multiplied by 2 as all are shared by two trapezia

(a) Complete the table above, giving values of y to 3 decimal places.

(2)

(b) Use the trapezium rule with all the y values in the completed table to find, to 2 decimal places, an estimate for

$$\int_{0.5}^{3} \left(1 - \log_{10}(\sin x)\right) dx$$

(3)

(c) Use your answer to part (b) to find an estimate for

$$\int_{0.5}^{3} \left(3 + \log_{10}(\sin x)\right) dx$$

(3)

(a) $y = 1 - \log_{10} \sin(1) = 1.07496$
$= 1.075$ (3 d.p.)

$y = 1 - \log_{10} \sin(2) = 1.04129...$
$= 1.041$ (3 d.p.)

Leave blank

Question 2 continued

(b) $A = \frac{1}{2}h\left[y_1 + 2(y_2 + y_3 + y_4) + y_5\right]$

$h = 1 - 0.5 = 0.5$

$A = \frac{1}{2}(0.5)\left[1.319 + 2(1.075 + 1.001 + 1.041 + 1.223) + 1.850\right]$

$= \frac{1}{4}(11.849) = 2.96225$

$= 2.96$ (2 d.p.)

(c) $\int_{0.5}^{3} 3 + \log_{10}\sin x \, dx = \int_{0.5}^{3} 4 - (1 - \log_{10}\sin x) \, dx$

$\boxed{\text{use rule : } \int f(x) \pm g(x)\,dx = \int f(x)\,dx \pm g(x)\,dx}$

$\int_{0.5}^{3} 4 - (1 - \log_{10}\sin x) \, dx$

$= \int_{0.5}^{3} 4 \, dx - \int_{0.5}^{3} 1 - \log_{10}\sin x \, dx$

REMEMBER!

$\boxed{\int x^n \, dx = \frac{x^{n+1}}{n+1}}$

$= 4x\Big]_{0.5}^{3} - 2.96$

$= 4(3) - 4(0.5) - 2.96$

$= 12 - 2 - 2.96 = 7.04$

Leave blank

Question 2 continued

ALTERNATIVE SOLUTION :

$$\int_{0.5}^{3} 1 - \log_{10}\sin x \, dx = \int_{0.5}^{3} 1 \, dx - \int_{0.5}^{3} \log_{10}\sin x \, dx$$

$$\int_{0.5}^{3} \log_{10}\sin x \, dx = \int_{0.5}^{3} 1 \, dx - \int_{0.5}^{3} 1 - \log_{10}\sin x \, dx$$

$$[x]_{0.5}^{3} - 2.96$$

$$3 - 0.5 - 2.96 = -0.46$$

$$\int_{0.5}^{3} 3 + \log_{10}\sin x \, dx = \int_{0.5}^{3} 3 \, dx + \int_{0.5}^{3} \log_{10}\sin x \, dx$$

$$= [3x]_{0.5}^{3} + (-0.46)$$

$$= 3(3) - 0.5(3) - 0.46$$

$$= 7.04$$

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Question 2 continued

(Total 8 marks)

Q2

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3. (i) Show that the following statement is **false**:

"$(n+1)^3 - n^3$ is prime for all $n \in \mathbb{N}$"

Natural numbers (1, 2, 3, 4...)

(2)

(ii) Given that the points $A(1, 0)$, $B(3, -10)$ and $C(7, -6)$ lie on a circle, prove that AB is a diameter of this circle.

(5)

(i) This proof is by counter-example.

- let $n = 1$

$(1+1)^3 - 1^3 = 7 \rightarrow$ prime

- let $n = 2$

$(2+1)^3 - 2^3 = 19 \rightarrow$ prime

- keep trying numbers until you find that:

- let $n = 5$

$(5+1)^3 - 5^3 = 91 = 7 \times 13$, not prime

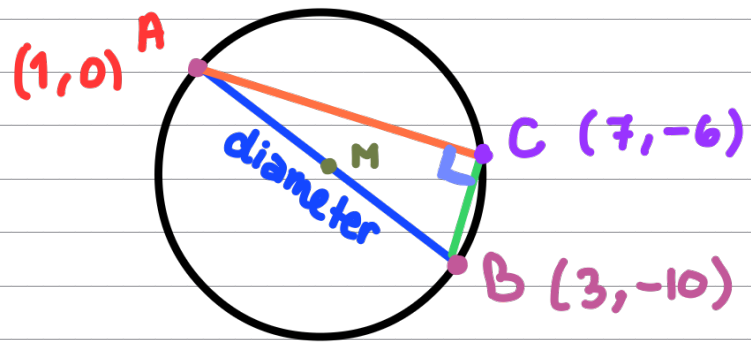
So statement is false.

(ii) If AB is a diameter, then according to circle theorems:

(1) triangle ABC is a right-angle triangle

(2) angle ACB is 90°

(3) If M is the midpoint AM = BM = CM = radius



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Question 3 continued

There are multiple approaches to this proof:

WAY 1:

$m_{AC} = \frac{-6-0}{7-1} = -1$ $\qquad$ $m_{BC} = \frac{-6-(-10)}{7-3} = 1$

$m_{AC} \times m_{BC} = -1 \times 1 = -1$

Hence, m_{AC} and m_{BC} are inverse reciprocals.

Therefore, angle ABC is a right angle, hence AB is a diameter (according to circle theorems)

WAY 2:

distance formula: $d = \sqrt{(x_1-x_2)^2+(y_2-y_1)^2}$

length AB $= \sqrt{(3-1)^2+(-10-0)^2} = \sqrt{104}$

length AC $= \sqrt{(7-1)^2+(-6-0)^2} = \sqrt{72}$

length BC $= \sqrt{(7-3)^2+(-6-(-10))^2} = \sqrt{32}$

$(AB)^2 = 104$
$(AC)^2 = 72$
$(BC)^2 = 32$

$104 = 72 + 32$
$a^2 = b^2 + c^2$
pythagoras theorem!

Hence, ABC is a right-angled triangle. Since AB is the longest, it is the hypotenuse of the triangle, hence AB is a diameter.

Leave blank

Question 3 continued

WAY 3 :

Find the midpoint of AB :

$$\text{midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$\text{midpoint}_{AB} = \left(\frac{1+3}{2}, \frac{0+(-10)}{2}\right)$$

$$= (2, -5)$$

Then : distance formula : $d = \sqrt{(x_1-x_2)^2+(y_2-y_1)^2}$

$MA = MB = \sqrt{(2-1)^2+(-5-0)^2} = \sqrt{26}$

$MC = \sqrt{(2-7)^2+(-5-(-6))^2} = \sqrt{26}$

As length from M to C is equal to the length from M to A and M to B, this means MC, MB, MA are all radii, and MA + MB = AB = 2(radius) = diameter.

WAY 4 :

midpoint = $(-2, 5)$ (shown in way 3)

$MA = \sqrt{26}$ (shown in way 3)

MA is a radius, so we can substitute it in the equation of the circle where

$$(x - x_{centre})^2 + (y - y_{centre})^2 = r^2$$

$x_c, y_c = (2, -5)$

$r = \sqrt{26}$

Leave blank

Question 3 continued

$(x-2)^2 + (y-(-5))^2 = (\sqrt{26})^2$

$(x-2)^2 + (y+5)^2 = 26$

substitute $C = (7, -6)$ into x, y

$(7-2)^2 + (-6+5)^2$

$= 5^2 + 1^2 = 26$

So since C satisfies the equation, AB is a diameter

Q3

(Total 7 marks)

Leave blank

4. **In this question you must show all stages of your working.**

Give your answers in fully simplified surd form.

Given that a and b are positive constants, solve the simultaneous equations

$$a - b = 8$$
$$\log_4 a + \log_4 b = 3$$

(6)

• Recall rules of logarithms :

$\log_a(b) + \log_a(c) = \log_a(b \times c)$

$\log_a(b) - \log_a(c) = \log_a\left(\frac{b}{c}\right)$

$\log_a(b) = c \longrightarrow a^c = b$

$\log_4 a + \log_4 b = \log_4(ab) = 3$

$ab = 4^3$

$ab = 64$

eq ① $ab = 64$

eq ② $a - b = 8 \longrightarrow a = 8 + b$

$b = a - 8$

$(8+b)(b) = 64$

$8b + b^2 = 64$

$1b^2 + 8b - 64 = 0$

OR

$a(a-8) = 64$

$a^2 - 8a = 64$

$1a^2 - 8a - 64 = 0$

QUADRATIC FORMULA:

$Ax^2 + Bx + C = 0$

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$b = \frac{-8 \pm \sqrt{8^2 - 4(1)(-64)}}{2(1)}$$

$$a = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(-64)}}{2(1)}$$

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Question 4 continued

$b = -4 + 4\sqrt{5}$

$= -4 - 4\sqrt{5}$ ✗

reject as b is positive

$a = 4 + 4\sqrt{5}$

$= 4 - 4\sqrt{5}$ ✗

reject as a is positive

So : $b = -4 + 4\sqrt{5}$

$a = 4 + 4\sqrt{5}$

Q4

(Total 6 marks)

Leave blank

5. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

Solve, for $-180° < \theta \leqslant 180°$, the equation

$$3\tan(\theta + 43°) = 2\cos(\theta + 43°)$$

(6)

$3\tan(\theta + 43°) = 2\cos(\theta + 43°) \rightarrow \tan\theta = \frac{\sin\theta}{\cos\theta}$

$\left(\frac{3\sin(\theta+43°)}{\cos(\theta+43°)} = 2\cos(\theta+43°)\right) \times \cos(\theta+43°)$

$3\sin(\theta+43°) = 2\cos^2(\theta+43°)$

Use identity : $\cos^2\theta + \sin^2\theta = 1$
$\cos^2\theta = 1 - \sin^2\theta$

$3\sin(\theta+43°) = 2(1-\sin^2(\theta+43°))$
$3\sin(\theta+43°) = 2 - 2\sin^2(\theta+43°)$

$2\sin^2(\theta+43°) + 3\sin(\theta+43°) - 2 = 0$
let $x = \sin(\theta+43°)$

$2x^2 + 3x - 2 = 0$
$(2x-1)(x+2) = 0$
$(2\sin(\theta+43°) - 1)(\sin(\theta+43°) + 2) = 0$

$\sin(\theta+43°) = \frac{1}{2}$
or!
$\sin(\theta+43)° = -2 \rightarrow$ reject as range of $\sin x$ is $-1 \leq x \leq 1$

WE HAVE TO SOLVE :

$\boxed{\sin(\theta+43°) = \frac{1}{2}, \quad -180°+43° \leq \theta+43° \leq 180°+43°}$

P 7 1 3 7 7 A 0 1 4 3 2

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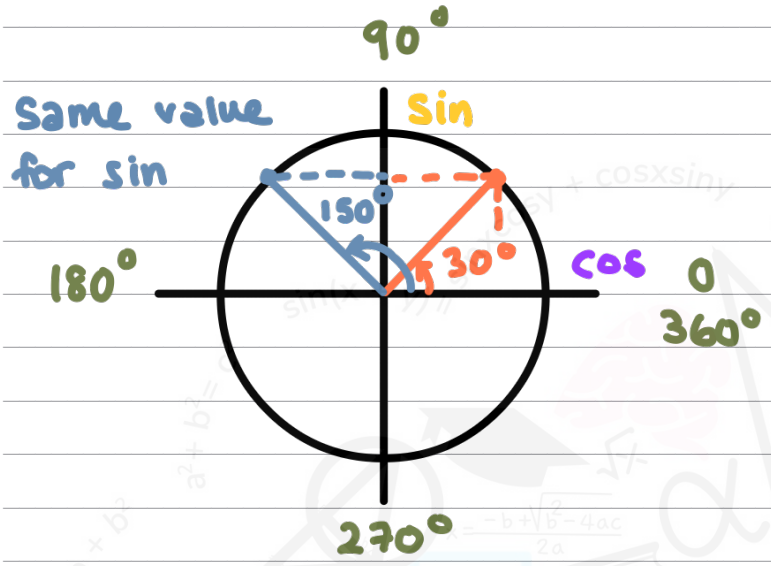
Question 5 continued

$\sin(\theta + 43^\circ) = \frac{1}{2}$

$(\theta + 43^\circ) = \sin^{-1}(1/2) = 30^\circ$

ON UNIT CIRCLE:



so: $(\theta + 43^\circ) = 180^\circ - 30^\circ = 150^\circ$

$\theta + 43^\circ = 30^\circ, \ 150^\circ$
$\quad -43^\circ \quad -43^\circ \quad -43^\circ$

$\theta = -13^\circ, 107^\circ$

Q5

(Total 6 marks)

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6. In a geometric sequence $u_1, u_2, u_3, \ldots$

- the common ratio is r
- $u_2 + u_3 = 6$ → $u_3 = 6 - u_2$
- $u_4 = 8$

(a) Show that r satisfies

$$3r^2 - 4r - 4 = 0$$

(3)

Given that the geometric sequence has a sum to infinity,

(b) find u_1

(3)

(c) find S_∞

(2)

(a) In geometric series :

$$u_1 + u_2 + u_3 + u_4 \ldots$$

r r r

$u_n = ar^{n-1}$

$u_2 = ar^{2-1} = ar$

$u_3 = ar^{3-1} = ar^2$

$u_4 = ar^{4-1} = ar^3$

$ar + ar^2 = 6$

$ar^3 = 8$

you can divide two equations !

$$\frac{ar + ar^2}{ar^3} = \frac{6}{8}$$

$$\frac{\cancel{a}r}{\cancel{a}r^3} + \frac{\cancel{a}r^2}{\cancel{a}r^3} = \frac{6}{8}$$

$$r^2 \times \left(\frac{1}{r^2} + \frac{1}{r}\right) = \left(\frac{6}{8}\right) \times r^2$$

Leave blank

Question 6 continued

$4 \times (1 + r) = \left(\frac{3}{4} r^2\right) \times 4$

$4 + 4r = 3r^2$

$3r^2 - 4r + 4 = 0$

ALTERNATIVE SOLUTION:

$u_4 = 8 \qquad u_3 = \frac{8}{r} \qquad u_2 = \frac{8}{r \times r} = \frac{8}{r^2}$

$r^2 \times \left(\frac{8}{r^2} + \frac{8}{r}\right) = (6) \times r^2$

$8 + 8r = 6r^2$

$[6r^2 - 8r - 8 = 0] \div 2$

$3r^2 - 4r - 4 = 0$

(b) S_∞ converges ONLY if: $-1 < r < 1$

$3r^2 - 4r - 4 = 0$

$(3r + 2)(r - 2) = 0$

$r = \frac{-2}{3}$ or $r = 2 \rightarrow$ reject, as $r > 1$

$ar^3 = 8$

$u_1 \left(\frac{-2}{3}\right)^3 = 8 \longrightarrow u_1 = -27$

(c) Use sum to infinity formula:

$S_\infty = \frac{a}{1 - r}$

$S_\infty = \frac{-27}{1 - \left(\frac{-2}{3}\right)} = -\frac{81}{5}$

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 8 marks)

Leave blank

7. $$f(x) = Ax^3 + 6x^2 - 4x + B$$

where A and B are constants.

Given that

- $(x + 2)$ is a factor of $f(x)$
- $\int_3^5 f(x)\,dx = 176$

find the value of A and the value of B.

(7)

① set the factor equal to zero :

$x + 2 = 0$

$x = -2 \longrightarrow$ so $f(-2) = 0$

② substitute $x = -2$ into $f(x) = 0$:

$f(x) = Ax^3 + 6x^2 - 4x + B$

$f(-2) = A(-2)^3 + 6(-2)^2 - 4(-2) + B = 0$

$-8A + 24 + 8 + B = 0$

$-8A + 32 + B = 0$

③ Find $\int_5^3 f(x)\,dx$

$f(x) = Ax^3 + 6x^2 - 4x + B$

REMEMBER!

$$\boxed{\int x^n\,dx = \frac{x^{n+1}}{n+1}}$$

$$\int_5^3 f(x)\,dx = \frac{A}{3+1}x^{3+1} + \frac{6}{2+1}x^{2+1} - \frac{4}{1+1}x^{1+1} + \frac{B}{0+1}x^{0+1} = 176$$

$$\left[\frac{A}{4}x^4 + 2x^3 - 2x^2 + Bx\right]_3^5 = 176$$

$$\left(\frac{A}{4}(5)^4 + 2(5)^3 - 2(5)^2 + B(5)\right) - \left(\frac{A}{4}(3)^4 + 2(3)^3 - 2(3)^2 + B(3)\right) = 176$$

P 7 1 3 7 7 A 0 2 0 3 2

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Question 7 continued

$\frac{625}{4}A + 250 - 50 + 5B - \frac{81}{4}A - 54 + 18 - 3B = 176$

$136A + 164 + 2B = 176$

$136A + 2B = 12$

(4) Solve simultaneous equations :

$-8A + 32 + B = 0$

(1) $B = 8A - 32$

(2) $136A + 2B = 12$

$136A + 2(8A - 32) = 12$

$136A + 16A - 64 = 12$

$152A = 76$

$A = \frac{1}{2}$

$B = 8(\frac{1}{2}) - 32 = -28$

Q7

(Total 7 marks)

Leave blank

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A curve has equation

$$y = 256x^4 - 304x - 35 + \frac{27}{x^2} \qquad x \neq 0$$

(a) Find $\frac{dy}{dx}$

(3)

$dy/dx = 0$

(b) Hence find the coordinates of the stationary points of the curve.

(5)

(a)

DIFFERENTIATION

$$\boxed{\begin{aligned} y &= ax^n \\ \frac{dy}{dx} &= nax^{n-1} \end{aligned}}$$

$$y = 256x^4 - 304x - 35 + 27x^{-2}$$

$$\frac{dy}{dx} = (4)256x^{4-1} - (1)304x^{1-1} + (-2)27x^{-2-1}$$

$$= 1024x^3 - 304 - \frac{54}{x^3}$$

(b) At stationary points, $\frac{dy}{dx} = 0$

$$1024x^3 - 304 - \frac{54}{x^3} = 0$$

$$x^3 \times \left(1024x^3 - 304 - \frac{54}{x^3}\right) = (0) \times x^3$$

$$1024x^6 - 304x^3 - 54 = 0$$

$$1024(x^3)^2 - 304(x^3)^1 - 54 = 0$$

This can be solved as a quadratic equation; factorise:

$$(8x^3 + 1)(64x^3 - 27) = 0$$

Leave blank

Question 8 continued

$\sqrt{x^3} = \sqrt{\frac{-1}{8}} \longrightarrow x = \frac{-1}{2}$

or $\sqrt{x^3} = \sqrt{\frac{27}{64}} \longrightarrow x = \frac{3}{4}$

Now, find y-coordinates :

$y = 256\left(\frac{-1}{2}\right)^4 - 304\left(\frac{-1}{2}\right) - 35 + 27\left(\frac{-1}{2}\right)^{-2}$

$= 241$

$y = 256\left(\frac{3}{4}\right)^4 - 304\left(\frac{3}{4}\right) - 35 + 27\left(\frac{3}{4}\right)^{-2}$

$= -134$

so coordinates are :

$\left(-\frac{1}{2}, 241\right)$ and $\left(\frac{3}{4}, -134\right)$

Q8

(Total 8 marks)

Leave blank

9. A scientist is using carbon-14 dating to determine the age of some wooden items.

The equation for carbon-14 dating an item is given by

$$N = k\lambda^t$$

where

- N grams is the amount of carbon-14 **currently** present in the item
- k grams was the **initial** amount of carbon-14 present in the item
- t is the number of years since the item was made
- λ is a constant, with $0 < \lambda < 1$

(a) Sketch the graph of N against t for $k = 1$

$N = \frac{1}{2}k$

(2)

Given that it takes 5700 years for the amount of carbon-14 to reduce to half its initial value,

(b) show that the value of the constant λ is 0.999878 to 6 decimal places.

(2)

Given that Item A

- is known to have had 15 grams of carbon-14 present initially
- is thought to be 3250 years old

(c) calculate, to 3 significant figures, how much carbon-14 the equation predicts is currently in Item A.

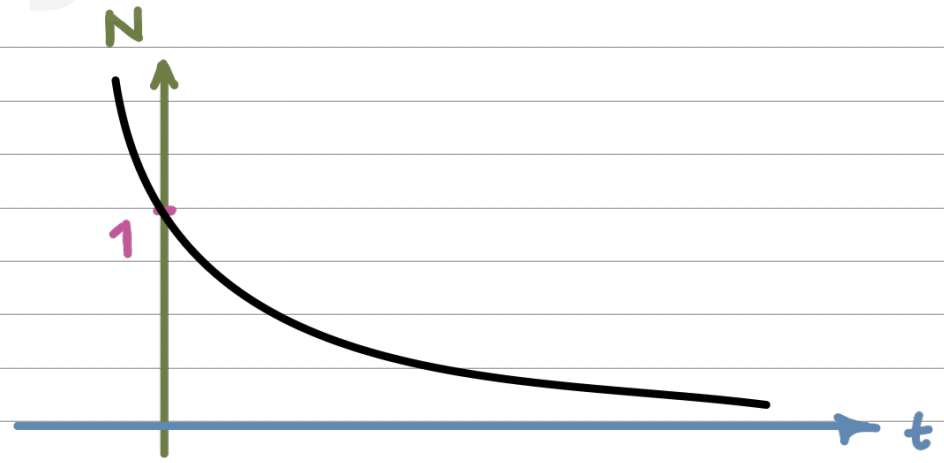
(2)

Item B is known to have initially had 25 grams of carbon-14 present, but only 18 grams now remain.

(d) Use algebra to calculate the age of Item B to the nearest 100 years.

(3)

(a) $N = (1)\lambda^t$



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Question 9 continued

(b) Substitute $N = \frac{1}{2}k$, $t = 5700$:

$N = k\lambda^t$

$\frac{1}{2}\cancel{k} = \cancel{k}\lambda^{5700}$

$\sqrt{\frac{1}{2}} = \sqrt{\lambda^{5700}}$

$\lambda = \left(\frac{1}{2}\right)^{5700} = 0.999878$ (6 d.p.)

(c) Substitute $k = 15$, $t = 32500$:

$N = 15(0.999878)^{32500}$

$= 10.1$ g (3 s.f.)

(d) Substitute $k = 25$, $N = 18$:

$18 = 25(0.999878)^t$

$\log(0.999878)^t = \log\frac{18}{25}$

$t = \frac{\log 18/25}{\log(0.999878)} = 2692.49...$

$= 2700$ years (nearest hundred)

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Q9

(Total 9 marks)

Leave blank

10. The circle C has centre $X(3, 5)$ and radius r

The line l has equation $y = 2x + k$, where k is a constant.

(a) Show that l and C intersect when

$$5x^2 + (4k - 26)x + k^2 - 10k + 34 - r^2 = 0$$

(3)

Given that l is a tangent to C,

(b) show that $5r^2 = (k + p)^2$, where p is a constant to be found.

(3)

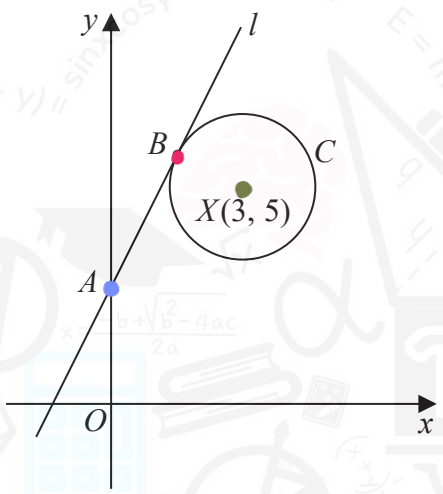

Figure 2

The line l

- cuts the y-axis at the point A
- touches the circle C at the point B

as shown in Figure 2.

Given that $AB = 2r$

(c) find the value of k

(6)

(a) ① write the equation of the circle :

$(x - x_{centre})^2 + (y - y_{centre})^2 = r^2$

$(x - 3)^2 + (y - 5)^2 = r^2$

$y_{circle} = y_l$ at intersection :

Leave blank

Question 10 continued

② substitute $y = 2x + k$ into circle eq.:

$(x-3)^2 + (2x+k-5)^2 = r^2$ → EXPAND!

$(x^2 - 6x + 9) + (4x^2 + 4(k-5)x + (k-5)^2) = r^2$

NOW, COMBINE LIKE TERMS:

$5x^2 + (-6 + 4k - 20)x + 9 + k^2 - 10k + 25 = r^2$

$5x^2 + (4k-26)x + k^2 - 10k + 34 - r^2 = 0$

(b) Since l is a tangent; line l and circle C intersect at only one point. Therefore, the equation we found in (a) has only one root.

Recall: $b^2 - 4ac$ → discriminant.

If $b^2 - 4ac > 0$ → multiple roots

→ $b^2 - 4ac = 0$ → one root

$b^2 - 4ac < 0$ → no roots

So:

$5x^2 + (4k-26)x + (k^2 - 10k + 34 - r^2) = 0$

$ax^2 + bx + c = 0$

$(4k-26)^2 - 4(5)(k^2 - 10k + 34 - r^2) = 0$

$16k^2 - 208k + 676 - 20k^2 + 200k - 680 + 20r^2 = 0$

$[-4k^2 - 8k - 4 + 20r^2 = 0] \div 4$

$-k^2 - 2k - 1 + 5r^2 = 0$

$5r^2 = (k^2 + 2k + 1)$ → $a^2 + 2ab + c^2$

$5r^2 = (k+1)^2$

Leave blank

Question 10 continued

(c) AXB is right-angled since $\angle ABX = 90°$.

So : $AB^2 + r^2 = AX^2$

$(AB)^2 = (2r)^2 = 4r^2$ → from question

$4r^2 + r^2 = 5r^2$

If $A = (0, k)$, $X = (3, 5)$, then :

$AX^2 = (3-0)^2 + (5-k)^2$ (distance formula)

$= 9 + (5-k)^2$

$AB^2 + r^2 = AX^2$

$5r^2 = AX^2$

$5r^2 = 9 + (5-k)^2$

from part (b) $(k+1)^2 = 9 + (5-k)^2$

$\cancel{k^2} + 2k + 1 = 9 + 25 - 10k + \cancel{k^2}$

$12k = 33$

$k = \frac{11}{4}$

ALTERNATIVE SOLUTION : (b+c)

(b) $m_{BX} = -\frac{1}{m_\ell} = \frac{-1}{2}$

$y = mx + c$

$5 = \frac{-1}{2}(3) + c$ → $c = \frac{13}{2}$

$y = \frac{-1}{2}x + \frac{13}{2}$ → eq. of radius

Leave blank

Question 10 continued

At B, $y_{radius} = y_e$

$$-\frac{1}{2}x + \frac{13}{2} = 2x + k$$

$$\frac{2}{5} \times \left(\frac{5}{2}x\right) = \left(\frac{13}{2} - k\right) \times \frac{2}{5}$$

$$x = \frac{13}{5} - \frac{2k}{5} = \frac{13-2k}{5}$$

$$y = -\frac{1}{2}\left(\frac{13-2k}{5}\right) + \frac{13}{2} = \frac{26+k}{5}$$

use equation of circle:

$$\left(\frac{13-2k}{5} - 3\right)^2 + \left(\frac{26+k}{5} - 5\right)^2 = r^2$$

$$\left(\frac{13-2k}{5}\right)^2 - \frac{6(13-2k)}{5} + 9 + \left(\frac{26+k}{5}\right)^2 - \frac{10(26+k)}{5} + 25 = r^2$$

$$\frac{(13-2k)^2}{25} - \frac{78-12k}{5} + \frac{(26+k)^2}{25} - \frac{260+10k}{5} + 34 = r^2$$

$$\left(\frac{(13-2k)^2 + (26+k)^2}{25} - \frac{338-2k}{5} + 34 = r^2\right) \times 25$$

$$169 - \cancel{52k} + 4k^2 + 676 + \cancel{52k} + k^2 - 5(338-2k) + 850 = 25r^2$$

$$1695 + 5k^2 - 1690 + 10k = 25r^2$$

$$[5k^2 + 10k + 5 = 25r^2] \div 5$$

$$k^2 + 2k + 1 = 5r^2$$

$$5r^2 = (k+1)$$

Leave blank

Question 10 continued

(c) We know:

$(AB)^2 = (2r)^2 = 4r^2 \longrightarrow$ from question

At B: $x = \frac{13-2k}{5}$ $\quad y = \frac{26+k}{5}$

$AB^2 = \left(\frac{13-2k}{5} - 0\right)^2 + \left(\frac{26+k}{5} - k\right)^2$

$4r^2 = \frac{(13-2k)^2}{25} + \left(\frac{26+k-5k}{5}\right)^2$

$= \frac{169-4k+4k^2}{25} + \frac{(26-4k)^2}{25}$

$4r^2 = \frac{169-52k+4k^2+676-208k+16k^2}{25}$

$25 \times (4r^2) = \left(\frac{845-260k+20k^2}{25}\right) \times 25$

$\frac{100r^2}{20} = \frac{845-260k+20k^2}{20}$

$5r^2 = \frac{169}{4} - 13k + k^2$

where $5r^2 = (k^2+2k+1)$

$\cancel{k^2} + 2k + 1 = \frac{169}{4} - 13k + \cancel{k^2}$

$15k = \frac{165}{4} \longrightarrow k = \frac{11}{4}$

Q10

(Total 12 marks)

TOTAL FOR PAPER IS 75 MARKS

END

P 7 1 3 7 7 A 0 3 2 3 2